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Abstract: We investigate the role of boundary conditions in gauge theories in AdS_4. The presence of the boundary can break the gauge symmetry consistently with AdS_4 isometries. We show that, as a consequence, the gauge bosons associated to the broken symmetries become massive at one loop. In particular chiral gauge theories such us the Standard Model are necessarily massive in AdS_4. We briefly discuss similarities with the Schwinger model and implications for CFTs in three dimensions.

Keywords: Gauge Symmetry, Global Symmetries, AdS-CFT Correspondence, Spontaneous Symmetry Breaking

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1 Introduction

Symmetry principles play an essential role in constraining the spectrum of quantum systems. In particular, in quantum field theory, massless particles are often understood as a consequence of symmetry. For spin 0 and spin 1/2 particles, supersymmetry and chiral symmetry are the relevant principles. For particles of spin 1 and higher, gauge symmetry is the relevant invariance. However gauge symmetry is not an ordinary symmetry but a redundancy in the parametrization of the dynamical variables. In the simplest situations this redundancy ensures the absence of the additional physical polarizations that are necessary to endow with mass particles with spin ≥ 1. This is the case of QED in 4D. In more general situations the gauge symmetry is not enough to forbid degrees of freedom acting as the extra polarizations, and then mass generation follows. In this case the gauge theory is said to be in the Higgs phase, and the additional polarizations are associated to Nambu-Goldstone (NG) bosons non-linearly realizing the gauge symmetry. In weakly coupled gauge theories the NG-bosons are elementary states. An example of that is given by the Standard Model with an elementary Higgs field. On the other hand if the interaction is sufficiently strong the role of NG-bosons can be played by composite states. This situation is realized for instance in technicolor models in 4D. Heuristically, a strong interaction among elementary constituents is needed in order to produce a NG pole out of a perturbative continuum spectrum in the current-current correlator.

The purpose of this paper is to illustrate how a Higgs mechanism involving a NG-boson composed by two elementary particles can arise at the perturbative level in gauge theories on AdS_4. The geometry of AdS_4 is crucial for this phenomenon to happen. On one side, in AdS_4 energy levels are discrete much like in finite volume (although AdS_4 has infinite volume), and therefore multi-particles states have a discrete mass spectrum. Moreover, since null geodesics reach the boundary of AdS_4 in finite time, bulk physics is crucially affected by boundary conditions. Indicating by $D(E, s)$ the one particle representations

of the AdS algebra [1] whose ground state has energy E and spin s, our basic point is the following. Measuring energies in units of the inverse AdS radius, a massless fermion ψ corresponds to $D(\frac{3}{2},\frac{1}{2})$. The two particle Hilbert space $\psi\otimes\psi$ then obviously contains the scalar representation $D(3,0)$. This corresponds to a derivatively coupled 4D scalar, a candidate NG-boson. Whether and how this scalar shifts under a gauge symmetry and thus causes the associated vector field to acquire a mass depends on the boundary conditions. Our basic remark is that charge breaking boundary conditions are compatible with AdS isometries. When charge breaking boundary conditions are imposed, the two fermion composite state is eaten by the bulk vector giving rise to a massive spin 1 multiplet

$$D\,(2,1)\oplus D\,(3,0)\longrightarrow D\left(\frac{3+\sqrt{1+4m^2}}{2},1\right) \tag{1.1}$$

As the mass is due to the mixing between 1- and 2-particle states, it will arise by considering the vector self-energy at 1-loop: $m^2\sim\frac{\alpha}{4\pi}$. A similar phenomenon can also arise for conformally coupled bulk scalars ϕ, that, depending on boundary conditions [2, 3], can be quantized as either $D(1,0)$ or $D(2,0)$. Again, in general we shall have $D(3,0)\subset\phi\otimes\phi$. In the case of scalars there is always the option to choose charge preserving boundary conditions. On the contrary, for chiral gauge theories the boundary necessarily breaks the gauge group to a subgroup of the maximal vector subgroup. So, for instance, in the Standard Model on AdS_4 even in the absence of an elementary Higgs field, the electro-weak vector bosons have a small mass.

This paper is organized as follows. In section 2 we introduce general boundary conditions for massless fermions and compute the 1-loop contribution to the vector boson mass. Furthermore we discuss the result for QED and for chiral gauge theories and illustrate how things change for massive fermions. In section 3 we include scalars and extend our result to supersymmetric gauge theories. We check that the resulting vector and gaugino masses are consistent with the Super-AdS algebra. In particular the so called 'anomaly mediated' gaugino mass is an essential contribution. Finally, in section 4 we discuss our results comparing to the mass generation in the Schwinger model in 1+1 dimensions and providing a holographic interpretation according to the AdS/CFT correspondence.

2 Mass generation

Let us consider a gauge theory with group G in AdS_4 space coupled to n massless Weyl fermions in a representation of G, generally reducible and anomaly free. The bulk action reads

$$\begin{aligned} S &= \int d^3x dz\, e\left[-\frac{1}{2}TrF^2-\frac{i}{2}(\bar\psi^i\bar\sigma^M D_M\psi_i+h.c.)\right]\\ D_M &= \partial_M+\omega_M+igA^a_M T_a \end{aligned} \tag{2.1}$$

where ω_M is the spin connection. Working in the Poncairé patch we take $e^A_M=L/z\,\delta^A_M$ and we follow all the conventions of [4]. In the massless limit the action is Weyl invariant so that it can be rescaled (classically) to half of flat space. It turns out that, for the

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computation we are interested in, we can easily bypass the complication associated to the breaking of Weyl rescaling by the UV regulator. A similiar approach was taken in ref. [5]. We will therefore perform all the computations using flat space variables.

In AdS the presence of a boundary at $z = 0$ requires that boundary terms are added to the action in order to make the variational problem well defined. These terms are in general described by a symmetric matrix B

$$\frac{1}{4}\int d^3x\, B_{ij}\,\psi^i\psi^j + h.c. \tag{2.2}$$

implying the following boundary conditions

$$\psi_i|_{z=0} = -iB^*_{ij}\sigma^3\bar{\psi}^j|_{z=0}. \tag{2.3}$$

The existence of non trivial ($\psi \neq 0$) solutions to the above equation requires that B is a symmetric unitary matrix. In the massless limit the free bulk matter lagrangian has a chiral symmetry U(n). However, since the boundary matrix B transforms under a chiral rotation as $B \to U^T BU$, it follows that the boundary breaks the symmetry to $O(n)$. This will necessarily break part of the gauge symmetry unless $G \subseteq O(n)$. The unbroken generators of G satisfy

$$T^*_a B + BT_a = 0 \tag{2.4}$$

i.e. they provide a real representation of the algebra. Note that such a general form of mass matrix would not be allowed in the bulk: 'explicit' breaking of the gauge symmetry in the bulk is equivalent to adding the corresponding elementary NG-bosons, in contradiction with the goal stated in the Introduction. In AdS space, however, the fields at the boundary are not dynamical and charge breaking conditions can be imposed. By eq. (2.3) there is no energy-momentum flow at the boundary, thus ensuring the compatibility with the isometries of AdS_4. This property distinguishes our set up from previous literature on mass generation on AdS_4, where transparent boundary conditions were imposed, corresponding to the presence of extra states (associated to a defect CFT) [6, 7]. In our set up only the gauge charge flows through the boundary. Indeed it must be stressed that, for chiral gauge theories, charge breaking at the boundary is mandatory, as the representation is complex and only a subgroup of G can be preserved. The necessity to break chirality in AdS_4 as a consequence of the relevance of the 3D boundary was first noticed in ref. [8], but the implications for chiral gauge theories where not investigated in that paper.

The above boundary conditions determine the fermion propagators to be

$$\langle\psi_{i\alpha}(X_1)\bar{\psi}_{j\dot{\beta}}(X_2)\rangle = \frac{i}{2\pi^2}\frac{(X_1-X_2)_M\sigma^M_{\alpha\dot{\beta}}}{[(X_1-X_2)^2+i\epsilon]^2}\,\delta_{ij}, \tag{2.5}$$

$$\langle\psi_{i\alpha}(X_1)\psi^\beta_j(X_2)\rangle = -\frac{1}{2\pi^2}\frac{(X_1-\tilde{X}_2)_M(\sigma^M\bar{\sigma}^3)^\beta_\alpha}{[(X_1-\tilde{X}_2)^2+i\epsilon]^2}\,B^*_{ij}. \tag{2.6}$$

where $\tilde{X} = (x, -z)$. Eqs. (2.5) and (2.6) are naturally associated to, respectively, direct propagation and propagation with one reflection at the boundary. The second equation

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implies the presence of a condensate in the bulk,

$$\langle \psi_i \psi_j(X) \rangle = \frac{1}{8\pi^2} \frac{B^*_{ij}}{z^3} \tag{2.7}$$

which spontaneously breaks the chiral U(n) global symmetry of the bulk action to $O(n)$.

We are now ready to compute the gauge boson mass at 1-loop. The self-energy due to the matter action decomposes into two contributions, from direct and reflected propagators.[1] Let us consider first the second contribution which is not ambiguous and does not require regularization. Defining the 1PI effective action as $\Gamma_{1PI} \in \int 1/2 A^a_M(X_1)\, \Pi^{MN}_{ab}(X_1, X_2) A_{Nb}(X_2)$ we have,

$$\begin{aligned} \Pi^{MN}_{R\,ab}(X_1, X_2) &= -i\, g^2 \kappa_{ab} \left(\frac{i}{2\pi^2}\right)^2 Tr\left[\sigma^M \bar\sigma^3 \sigma^Q \bar\sigma^N \sigma^P \bar\sigma^3\right] \frac{(X_1 - \tilde X_2)_P (X_1 - \tilde X_2)_Q}{(X_1 - \tilde X_2)^8} \\ &= -i\, \frac{g^2 \kappa_{ab}}{2\pi^4} \left[\frac{\tilde\eta^{MN}}{(X_1 - \tilde X_2)^6} - \frac{2 (X_1 - \tilde X_2)^M (X_1 - \tilde X_2)^P \tilde\eta^N_P}{(X_1 - \tilde X_2)^8} \right] \end{aligned} \tag{2.8}$$

where $\tilde\eta_{MN} = Diag(-1, 1, 1, -1)$ and $\kappa_{ab} = Tr[B T_a B^* T_b^*]$. As expected Π^{MN}_R is transverse, $\partial_{X_1^M} \Pi^{MN}_R(X_1, X_2) = \partial_{X_2^N} \Pi^{MN}_R(X_1, X_2) = 0$, guaranteeing that the effective action is gauge invariant in the bulk. Indeed the self-energy can be written as

$$\Pi^{MN}_{R\,ab}(X_1, X_2) = i \frac{2}{3} \frac{g^2 \kappa_{ab}}{(4\pi^2)^2} \tilde\eta^M_P \left(\frac{\partial}{\partial X_{2N}} \frac{\partial}{\partial X_{2P}} - \eta^{PN} \Box_2 \right) \frac{1}{(X_1 - \tilde X_2)^4} \tag{2.9}$$

To extract the photon mass we proceed as in [5] and compute the 1-loop corrected 'equations of motion' associated to the 1PI effective action. We suppress the non abelian indices as they factor out. By integrating twice by parts the 1-loop contribution to the equations of motion, we obtain

$$\begin{aligned} EQM_R &= \int d^4 X_2\, \Pi^{MN}_R(X_1, X_2) A_N(X_2) \\ &= -i \frac{2}{3} \frac{g^2 \kappa}{(4\pi^2)^2} \tilde\eta^{MP} \int d^4 X_2 \frac{1}{(X_1 - \tilde X_2)^4} \frac{\partial}{\partial X_{2Q}} F_{QP}(X_2) \\ &\quad -i \frac{2}{3} \frac{g^2 \kappa}{(4\pi^2)^2} \tilde\eta^{MP} \int d^3 x_2 \left[\eta_{P3} \frac{\partial}{\partial X_{2Q}} \frac{1}{(X_1 - \tilde X_2)^4} A_Q(X_2) - \frac{\partial}{\partial z_2} \frac{1}{(X_1 - \tilde X_2)^4} A_P(X_2) \right] \Big|_{z_2 = 0} \\ &\quad -i \frac{2}{3} \frac{g^2 \kappa}{(4\pi^2)^2} \tilde\eta^{MP} \int d^3 x_2 \left[\frac{1}{(X_1 - \tilde X_2)^4} F_{3P}(X_2) \right] \Big|_{z_2 = 0} \end{aligned} \tag{2.10}$$

At leading order the contribution to the mass can be derived by evaluating the equations of motion on massless solutions. We find it convenient to use solutions that satisfy the gauge condition

$$\partial^M \left[\frac{A_M}{z^2} \right] = 0 \tag{2.11}$$

[1] We will not compute the contribution from gauge loops as, first of all, it is independent of the matter contribution and absent is abelian theories. Secondly, we could not identify any two particle state playing the role of the NG-boson in the two vector channel.

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which corresponds, in our choice of coordinates, to the general covariant Lorentz gauge condition $D^M A_M = 0$. For the same reasons explained in [5], we also need to impose Hartle-Hawking boundary conditions at the horizon $z = \infty$ of the Poincaré patch. A physical (not pure gauge) set of solutions of the massless equations in Lorentz gauge is

$$A_3 = 0, \qquad A_\mu = e^{i(p_\nu x^\nu + |p|z)}\epsilon_\mu \qquad\qquad \epsilon_\mu p^\mu = 0, \qquad\qquad \mu = 0,1,2 \tag{2.12}$$

so we will compute the action of the self-energy on these functions and show it acts like a local mass term. Notice that in the massless case the Lorentz gauge leaves one residual gauge degree of freedom. This additional polarization is pure gauge in the massless case but becomes the physical 3rd polarization in the massive case. We have checked that the self energy acts like the same local mass term also on this additional polarization, for which computations are slightly more involved.

The bulk contribution in the second line of (2.10) is zero by the tree level equations of motion. The boundary terms give rise to a mass term. To see this, the first term in the third line of eq. (2.10) vanishes due to $\epsilon_\mu p^\mu = 0$ while the second gives

$$i\frac{2}{3}\frac{g^2\kappa}{(4\pi^2)^2}\frac{\partial}{\partial z_1}\int d^3x_2 \frac{1}{[(x_1-x_2)^2+z_1^2+i\epsilon]^2}\, e^{i\,p_\nu x_2^\nu}\epsilon_\mu = -\frac{2}{3}\frac{g^2\kappa}{(4\pi)^2}\left[\frac{1}{z_1^2} - i\frac{|p|}{z_1}\right]\, e^{i(p_\nu x_1^\nu + |p|z_1)}\epsilon_\mu. \tag{2.13}$$

From the fourth line we obtain

$$-i\frac{2}{3}\frac{g^2\kappa}{(4\pi^2)^2}\int d^3x_2 \frac{i|p|}{[(x_1-x_2)^2+z_1^2+i\epsilon]^2}\, e^{i\,p_\mu x_2^\mu}\epsilon_\mu = -\frac{2}{3}\frac{g^2\kappa}{(4\pi)^2}\left[i\frac{|p|}{z_1}\right]\, e^{i(p_\nu x_1^\nu + |p|z_1)}\epsilon_\mu. \tag{2.14}$$

Combining the two we see that Π_R^{MN} acts as a mass term,

$$\delta m^2_{ab}|_{reflected} = \frac{2}{3L^2}\frac{g^2\kappa_{ab}}{(4\pi)^2} \tag{2.15}$$

The contribution to the self-energy from the direct propagators is more subtle, as it requires UV regulation. However its computation can be bypassed. Indeed the direct contribution to the vector boson self-energy is proportional to $\mathrm{Tr}[T_a T_b]$, and independent of the boundary matrix B. So we simply have

$$\delta m^2_{ab}|_{direct} = c\frac{1}{L^2}\frac{1}{(4\pi)^2}\mathrm{Tr}[T_a T_b]\,. \tag{2.16}$$

with c a numerical coefficient. The request that the sum of reflected and direct contributions to the vector mass vanish in the charge preserving case, $BT_a = -T_a^* B$, fixes $c = 2/3$. In the end the total contribution to the vector mass is

$$m^2_{ab} = \frac{2}{3L^2}\frac{g^2}{(4\pi)^2}\,\mathrm{Tr}[BT_a B^* T_b^* + T_a T_b] \tag{2.17}$$

As a matter of fact we have also performed an explicit computation of the direct contribution to the self-energy confirming the above result. The matrix in eq. (2.17) is easily proven to be positive semi-definite. Indeed $T \to B^* T^* B \equiv T_B$ is an orthogonal transformation

with respect to the natural metric $\mathrm{Tr}T_1T_2 \equiv \langle T_1|T_2\rangle$ on the space of hermitean matrices. Defining $T = \alpha_a T_a$ we have then

$$\alpha_a\alpha_b m^2_{ab} \propto \langle T|T_B\rangle + \langle T|T\rangle \geq 0 \tag{2.18}$$

with $\alpha_a\alpha_b m^2_{ab} = 0$ occurring if and only if $T = -B^*T^*B \equiv T_B$. Therefore the gauge bosons associated to the broken generators acquire positive mass^2. This is the main result of our paper.

2.1 QED

To be concrete, let us consider QED coupled to two massless Weyl fermions of opposite charges. In this case the general boundary matrix depends on three real parameters and can be conveniently parameterized as,

$$B = \begin{pmatrix} i\lambda\, e^{2i\phi_1} & \sqrt{1-\lambda^2}\, e^{i(\phi_1+\phi_2)} \\ \sqrt{1-\lambda^2}\, e^{i(\phi_1+\phi_2)} & i\lambda\, e^{2i\phi_2} \end{pmatrix} \tag{2.19}$$

with $0 \leq \lambda \leq 1$. From eq. (2.17) the mass of the photon is $m^2_\gamma = g^2\,\lambda^2/(6\pi^2 L^2)$ and does not depend on $\phi_{1,2}$. Indeed the bulk Lagrangian of massless QED is classically invariant under a $\mathrm{U}(1)\times\mathrm{U}(1)$ symmetry corresponding to electric charge and chiral symmetry. Due to this bulk symmetry, the two phases can be eliminated by the a field redefinition that does affect the bulk lagrangian. Note however that by adding bulk operators that break the chiral symmetry (for instance 4 fermion interactions) the combination $\phi_1+\phi_2$ becomes observable, while $\phi_1-\phi_2$ can always be set to zero by charge rotations. In the end, the physically relevant parameters are in general 2, with the vector boson mass taking values in a fixed range $0 \leq m^2_\gamma L^2 \leq g^2/6\pi^2$. Notice that in the general case of many fermions with charge matrix Q, the maximal value the vector mass can attain is proportional to $\mathrm{Tr}Q^2$, that is the same combination of charges that controls the 1-loop β-function.

2.2 Chiral theories

As we mentioned earlier, when the matter fields are in a complex representation of G, i.e. the gauge theory is chiral, no boundary conditions that preserve the full gauge symmetry can be chosen and therefore some of the gauge bosons necessarily become massive. In this case the maximal symmetry that can be preserved is the maximal vectorial subgroup of G.

As an example let us consider an SU(5) gauge theory with fermions in the $5+\bar{10}$ representation. The maximal vector subgroup is SU(4) under which the fermions decompose as $1\oplus 4\oplus\bar{4}\oplus 6$. The broken generators transform in the $4\oplus\bar{4}\oplus 1$ representation of SU(4) and the associated gauge bosons will acquire mass. For the Standard Model the maximal vectorial subgroup is instead $\mathrm{SU}(3)\otimes\mathrm{U}(1)$. Let us consider a quark doublet. In order the preserve $\mathrm{SU}(3)\otimes\mathrm{U}(1)$ the boundary conditions give rise to the following condensates,

$$\begin{aligned} < dd_c > &= \frac{3}{8\pi^2 z^3} \\ < uu_c > &= \frac{3}{8\pi^2 z^3} \end{aligned} \tag{2.20}$$

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The pattern of chiral symmetry breaking is identical to the one in QCD and the NG-bosons associated to this breaking become the longitudinal components of the W, Z bosons. In fact, as in the Standard Model, due to the unbroken SU(2) "custodial" symmetry rotating up and down quarks, we have $m_W^2/m_Z^2 = \cos^2\theta_W$ with θ_W the Weinberg angle. The same conclusion holds in the lepton sector if there exists a right-handed neutrino. Otherwise there will be necessarily a ν^2 condensate which breaks custodial symmetry (and lepton number) and modifies the previous ratio. Notice, finally, that in the SU(5) and $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ examples the vector boson mass at the point of maximal symmetry is fixed.

2.3 Bulk mass

For vector theories we can add a bulk mass for the fermions. In this case the boundary conditions consistent with AdS_4 invariance are more restricted: the bulk mass gives a discrete set of possibilities for the scaling of the solution at the boundary. This follows from the fact that the boundary matrix and the bulk mass matrix must be simultaneously diagonalizable. As discussed in [5], for a Weyl spinor of mass $mL \geq 1/2$ (we assume without loss of generality m to be real and positive) the bulk action already implies the boundary condition uniquely. The resulting one particle Hilbert space corresponds to the $D(\frac{3}{2}+mL, \frac{1}{2})$ representation. In the case of QED this unique boundary condition is, not surprisingly charge preserving, implying a massless photon. As a quick check of that, notice that the tensor product $D(\frac{3}{2}+mL, \frac{1}{2})\otimes D(\frac{3}{2}+mL, \frac{1}{2})$ does not contain the NG representation $D(3,0)$. In the region $mL < 1/2$, analogously to the scalar double quantization [2, 3], two inequivalent boundary conditions are allowed for each Weyl fermion. In the presence of multiple Weyl fermions with the same mass, the above discrete set of boundary conditions can be folded by a rotation among the fields of equal mass. If one performs this exercise for QED coupled to a massive Dirac fermion, one finds 3 inequivalent possibilities for the matrix B describing the boundary condition

$$B_\pm = \pm \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad B_0 = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix} . \tag{2.21}$$

The choices B_+ and B_- preserve charge and correspond to a Dirac fermion in respectively $D(\frac{3}{2}+mL, \frac{1}{2})$ and $D(\frac{3}{2}-mL, \frac{1}{2})$. The choice B_0 breaks charge maximally. The one particle Hilbert space corresponds to a direct sum of Majorana spinors $D(\frac{3}{2}+mL, \frac{1}{2})\oplus D(\frac{3}{2}-mL, \frac{1}{2})$. In this case, as expected, a Goldstone multiplet appears in the two particle Hilbert space. The angle ϕ has no physical consequences as it can be eliminated by a gauge rotation. Not surprisingly, since chiral symmetry is broken by the bulk mass, the freedom in $\phi_1 + \phi_2$ has disappeared with respect to eq. (2.19). But, less trivially, the parameter λ controlling charge breaking is now 'quantized' to be either 0 or 1. Contrary to the massless case, the action is not Weyl invariant so that flat space formulae cannot be used to compute the vector mass and a genuine AdS_4 computation is required. We of course expect the gauge boson to be massive also in this case with a mass that goes to zero as $mL \to 1/2$.

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3 Supersymmetry

The previous results can be easily extended to the supersymmetric version of the theory. Supersymmetric QED in AdS_4 with charge preserving boundary conditions was studied in [5] so we will consider this case. In that paper it was shown that an ultraviolet counter-term, the anomaly mediated gaugino mass [9], was required to cancel an infrared contribution associated to the $R-$symmetry breaking from the boundary in order to leave the gaugino massless as demanded by supersymmetry.

In general, supersymmetry also allows for charge breaking boundary conditions with the scalars in a chiral multiplet aligned with the fermions

$$\phi_i|_{z=0} = B^*_{ij}\phi^{j*}|_{z=0}. \tag{3.1}$$

For zero mass term in the superpotential the scalars are also conformally coupled so the full action can again be rescaled to half of flat space. Up to a numerical factor, the contribution to the photon self energy from the scalar loop is identical to the one of the fermions. In fact this loop is proportional to the $\beta-$function of the theory. Since a complex scalar contributes one half of a Weyl spinor in the $\beta-$function we find that the photon mass is 3/2 of eq. (2.17). In terms of AdS_4 representations (see [1]), since for vector fields $m_1^2L^2 = E(E-3)+2$ this corresponds to

$$D\left(2+\frac{g^2(\kappa+2)}{(4\pi)^2},1\right) \qquad \kappa \equiv \mathrm{Tr}BQB^*Q^* \tag{3.2}$$

By imposing charge breaking boundary conditions the infrared contribution to gaugino mass will not cancel exactly the anomaly mediated one, which is independent of the boundary conditions. Repeating the same steps as in [5] one finds

$$m_\lambda = \frac{g^2(k+2)}{(4\pi)^2 L} \tag{3.3}$$

This corresponds to the AdS representation ($m_{1/2}L = E - 3/2$),

$$D\left(\frac{3}{2}+\frac{g^2(k+2)}{4\pi^2},\frac{1}{2}\right) \tag{3.4}$$

Recalling that a massive vector multiplet decomposes into the following representations

$$D\left(E_0,\frac{1}{2}\right)\oplus D\left(E_0+\frac{1}{2},0\right)\oplus D\left(E_0+\frac{1}{2},1\right)\oplus D\left(E_0+1,\frac{1}{2}\right), \tag{3.5}$$

we conclude that the photon and gaugino acquire masses as demanded by supersymmetry with $E_0 = \frac{3}{2}+\frac{g^2(k+2)}{4\pi^2}$. Notice that $D(E_0+\frac{1}{2},0)$ and $D(E_0+1,\frac{1}{2})$ correspond to purely two-particle states. In order to check the satisfaction of the algebra for these states we would have to study the Källen-Lehmann decomposition of the current correlators. We have not performed this additional computation.

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4 Discussion

The mass generation described in this paper presents some similarities with the Schwinger model in 1+1 dimensions [10]. In that case, even before turning on any interaction, the peculiar kinematics of 2D space-time implies the presence of a normalizable massless state composed of a fermion and an anti-fermion. When the gauge coupling is turned on, the vector field acquires a mass at 1-loop by 'eating' that massless bound state. In our set up it is the kinematics of AdS_4 that guarantees the presence of a normalizable massless scalar state $D(3,0)$. Depending on the boundary conditions, when the gauge coupling is turned on, the vector boson may eat the bound state and become massive. The technical difference between the two cases lies in the fact that in the Schwinger model there is no charge breaking scalar condensate made up of two fermion fields. This is due to Coleman's theorem [11] which establishes the absence of spontaneous symmetry breaking in 2D field theory. In our $4D$ example, instead, massless ($D(3,0)$) scalar fields have a moduli space of expectation values, determined by boundary conditions. In this sense the mass generation in AdS_4 is qualitatively similar to technicolor theories where a condensate is responsible for the breaking. However the distinction between the case with and without condensate is just technical, since in gauge theories the only observable operators are gauge invariant ones. From a gauge invariant viewpoint the story is the same in the two cases: starting from a free theory with a massless elementary vector and a massless scalar 'bound' state, a massive vector emerges when the interaction is turned on.

Finally we would like to comment on our results from the standpoint of the AdS/CFT correspondence [12]. The phenomenon we have studied corresponds to turning on double-trace (marginal) deformations in the dual CFT_3 (see also [14] for related work). A (complex) 4D Weyl fermion ψ quantized to give the $D(E,\frac{1}{2})$ representation corresponds to a (real) 3D fermionic operator Ψ of scaling dimension E. In particular for a massless 4D fermion the dimension of Ψ is $\frac{3}{2}$. The scalar operator $\mathcal{O}=\Psi\Psi$ has dimension 3 (in the large N limit) and represents a double trace marginal deformation. By simple OPE analysis (like for instance done in ref. [13]) one is indeed convinced that $\lambda\Psi\Psi$ is exactly marginal. This generalizes to the case of a number n of fermions ψ_i ($i=1,\ldots,n$), corresponding to CFT_3 operators Ψ_i. In that case the most general marginal deformation $\Delta\mathcal{L}_{CFT}=\lambda_{ij}\Psi_i\Psi_j$ is associated to $(n^2+n)/2$ real parameters. This precisely corresponds to the number of real free parameters in the boundary matrix B_{ij} on the AdS_4 side, although to derive the mapping between the two sets some work is needed [14]. When the AdS_4 gauge group $G\subseteq O(n)$, the corresponding CFT_3 can have global symmetry G. The most general marginal parameters λ_{ij} will in general break G to a subgroup H. Corresponding to the vector bosons getting a 1-loop mass in the bulk, the CFT_3 currents in G/H will acquire anomalous dimensions of order $\alpha/4\pi\sim 1/N^2$. The anomalous dimensions will also depend on the deformation parameters and vanish continuously at the points of the λ_{ij} space where G invariance is restored. On the other hand, when $G\not\subseteq O(n)$, corresponding to a chiral gauge theory in AdS, the dual CFT will be at best invariant under $G'=G\cap O(n)$. In that case the currents in G/G' will acquire a non-zero $O(1/N^2)$ anomalous dimension over the entire moduli space.

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It is instructive to see in more detail how things work in the simple QED example, sketching the dual picture of the discussion in subsection 2.1. The dual CFT contains two fermionic operators that can be packaged into one complex field $\Psi = \Psi_1 + i\Psi_2$, with charge one under the global U(1) symmetry. The most general double-trace deformation is then

$$\Delta\mathcal{L}_{CFT} = \lambda_1 \Psi^* \Psi + \left(\lambda_2 e^{i\theta} \Psi\Psi + \text{h.c.}\right) \tag{4.1}$$

again described by 3 real parameters (λ_1, λ_2, θ). The phase θ obviously has no physical consequence, as it can be eliminated by a U(1) rotation. (This remark applies more generally to the previous discussion: the physically relevant λ_{ij} are determined by modding out by G.). We are left with two physical parameters λ_1 and λ_2. When $[\Psi] = \frac{3}{2}$ these parameters are exactly marginal. Of the two, λ_2 explicitly breaks U(1) and must clearly be consequential: the dual picture of our 4D computation is that the current acquires an anomalous dimension. The parameter λ_1 does not break any obvious symmetry of the CFT. What happen here is clarified by the AdS_4 picture. λ_1 is basically associated to the $\phi_1 + \phi_2$ phase in eq. (2.19): as long as the bulk theory is invariant under the *global* chiral symmetry, this phase can be eliminated and bulk physics remains the same. The CFT interpretation of this phenomenon should be that when λ_1 is turned on there exists a field redefinition by which the deformed CFT is shown to be exactly equivalent to the original one. This peculiarity should correspond to the 3D reflection of 4D global chiral symmetry [14]. Of course one could conceive a 4D theory where, by tuning, the fermions are massless while chirality is broken by other interactions, for example Yukawa or 4-fermion interactions. In that situation λ_1 would parameterize an inequivalent moduli space of CFTs. Finally, let us consider the case where a bulk mass is turned on. At the charge preserving points, Ψ has either dimension $\frac{3}{2} + m$ or $\frac{3}{2} - m$. Consider indeed the second possibility. Eq. (4.1) is now a relevant deformation. According to the 4D picture, this deformation makes the CFT flow to discrete set of inequivalent fixed points. For $\lambda_1 \neq 0$ and $\lambda_2 = 0$ electric charge is conserved and the theory flows to the other possible charge preserving quantization the one where there is a fermion operator Ψ' of dimension $\frac{3}{2} + m$. Instead for $\lambda_1 = 0$ and $\lambda_2 \neq 0$ the flow will lead to a new CFT where the current has a definite $O(1/N^2)$ anomalous dimension.

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